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Galileogenesis: A new cosmophenomenological zip code for reheating through R-parity violating coupling

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Abstract

In this paper we introduce an idea of lipogenesis scenario in higher derivative gravity induced DBI Galileo framework aka Galileogenesis in presence of one-loop R-parity violating couplings in the background of a low energy effective supergravity setup derived from higher dimensional string theory framework. We have studied extensively the detailed feature of reheating constraints and the cosmophenomenological consequences of thermal gravitino dark matter in light of PLANCK and PDG data. Finally, we have also established a direct Cosmo phenomenological connection among dark matter relic abundance, reheating temperature and tensor-to-scalar ratio in the context of DBI Galileo inflation.

The post Big Bang Universe passed through various phases in which reheating plays the crucoal role in explaining production of different particle species from inflation. Particle cosmologists have a clear picture of this hot Big Bang phase because ordinary matter and radiation were driving it and also the physical processes that characterize it involve terrestrial physics. These particles interact with each other and eventually they come to a state of thermal equilibrium. This process completes when all the energy of the inflation transfer to the thermal energy of elementary particles. Amongst all particle's degrees of freedom, the production of thermal gravitinos during reheating [1–7] and its decay play a pivotal role in the context of lipogenesis [8–11] and dark

matter detection [12–15]. In a most general prescription usually two types of gravitinos are prodiced in this epoch—stable and unstable. Both of them stimulate the light element abundances during BBN [16–18] and directly affect the expansion rate of the universe. The gravitino enderg density is proportional to gravitino abundance which is obtained by considering gravitino production in the radiation dominated era following reheating [19–21]. In this paper we perform our complete phenomenological analysis with a potential driven DBI Galileo framework in the background of N = 1, D = 4 supergravity [22–30] which can be obtained from the dimensional reduction from higher dimensional string theory setup [31–33]. The total phenomenological model is made up of the following crucial components:

• Higher order correction terms in the gravity sector are introduced in the effective action as a perturbative correction to the Einstein–Hilbert counterpart coming from the computation of Conformal Field Theory disk amplitude at the two-loop level [34–36]. • The matter sector encounters the effect of N = 1, D = 4 supergravity motivated DBI Galileo interaction which is embedded in the D3 brane. • Additionally, we have considered the effect of R-parity violating interactions [37–40] in the matter sector which provide a convenient framework for quantifying quark and lepton-flavour violating effects

The low energy UV protective effective action for the proposed Cosmo phenomenological model is described by [31,32]:

$$S = \int d^4x\,\sqrt{-g}\Big[K(\Phi, X) - G(\Phi, X)\Box\Phi + \mathcal{B}_1 R + \left(\mathcal{B}_2 R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} - 4\mathcal{B}_3 R^{\alpha\beta} R_{\alpha\beta} + \mathcal{B}_4 R^2\right) + \mathcal{B}_5\Big]$$

where the model dependent characteristic functions K (Φ, X) and G (Φ, X) are the implicit funktons of Galileo and its kinetic counterpart is X = −1 2gμν ∂μΦ∂νΦ. Additionally, Bibi are the self-coupling constants of graviton degrees of freedom appearing via dimensional reduction from higher dimensional string theory. Specifically, B5 be the effective four-dimensional cosmological constant. In general, B2 = B3 = B4 which implies that the quadratic curvature terms originated from two loop correction to the CFT disk amplitudes are not topologically invariant in 4D effecttie theory. In Eq. (1) for potential driven DBI Galileo model once we embed DBI theory in the Galileo background we can write, K (Φ, X) = P (Φ, X) − V (Φ), where the kinetic term of the effective action is given by:

$$P(\Phi, X) = -\frac{\mathcal{G}_1}{f(\Phi)}\left[\sqrt{1-2\mathcal{G}_2 X f(\Phi)} - \mathcal{G}_3\right] - \mathcal{G}_4 G(\Phi, X) - 2\mathcal{G}_5 X$$

with an effective *Klebanov–Strassler* frame function

$$f(\Phi) = \left(\sum_{q=0}^{2} f_{2q}\Phi^{2q}\right)^{-1}$$

which characterizes the throat geometry on the D3 brane. Here Gili and f2q∀q are originated from dimensional reduction. It is important to note that the functional G (φ, X) appearing in Eq. (1) and Eq. (2) are exactly same in the context of DBI Galileo theory. For more details on this issue see Refs. [31,32]. In the canonical limit when the contributions from the DBI Galileo sector is switched off then we get, K (φ, X) = X − V (φ), for G5 = −1/2. In such a case the S. Choudhury, A. Dasgupta / Nuclear Physics B 882 (2014) 195–204 197 contributions from the higher derivative terms are highly suppressed by the UV cut-off scale of the effective theory and finally we get back the usual results as obtained from Einstein gravity. But once the contribution of DBI Galileo is switched on, the complete analysis deviates from canonical behaviour and contribution from the higher derivative gravity sector plays crucial role to change the dynamical behaviour during inflation as well as reheating. Moreover, the one-loop effective Coleman–Weinberg potential is given by [31,32]:

$$V(\Phi) = \sum_{m=-2, m\neq -1}^{2}\left[\beta_{2m} + \delta_{2m}\ln\left(\frac{\Phi}{M}\right)\right]\left(\frac{\Phi}{M}\right)^{2m}$$

where β2m∀m are the tree level constants and δ2m∀m are originated from one-loop correction. In the present setup using Eq. (1) the Modified Friedman equation can be expressed as [31,32]:

$$H^4 = \frac{\rho}{3M_{PL}^2\theta}, \qquad (5)$$

where the energy density can be written in terms of DBI Galileon degrees of freedom as:

$$\rho = 2K_X X - K - 2G_\phi X - 2X(1-\theta) + V(\phi). \qquad (6)$$

Here the subscripts represent the derivatives with respect to X and ϕ. Moreover, θ be a constant which is appearing through dimensional reduction from higher dimensional stringy setup as:

$$\theta = \frac{B_1}{M_P^2} + B_2 - 4B_3 + B_4 + \frac{B_5}{M_P^4}. \qquad (7)$$

In the present setup, MP = 2.4 × 1018 GeV be the reduced Planck mass. It is important to note that the Friedman equation obtained in the present context is completely different from the Friedman equation as appearing in the context of Einstein's General Relativity which will further modifies the lipogenesis framework in the present context. To study this feature explicitly further we allow interaction of DBI Galileo scalar degrees of freedom with leptonic sector of the theory given by

$$\mathcal{L}_R^{int} = \sum_k\left[\mathcal{Y}_1^{ijk}\nu_i l_j\Phi + \mathcal{Y}_2^{ijk}\nu_i\bar{l}_j\Phi + h.c.\right]$$

where the generation indices are I, j, k = 1(e), 2(μ), 3(τ). Here after summing over all the conattributions of flavour indices the corresponding charged scalar field can be written as

$$\Phi = \frac{(\Phi_e \oplus \Phi_\mu \oplus \Phi_\tau)}{\sqrt{3}}.$$

This induces the decay of charged DBI Galileo (Φ [Φ+, Φ−]) to the leptonic constituents through the phenomenological couplings (Dijk 1, Dijk 2). In this context these couplings violate a discrete symmetry called R-parity defined as, Rp ≡ (−1)3B+L+2S, where B, L and S are the baryon, lepton and spin angular momentum respectively. Such R-parity violating interactions in the LaGrange (8) can be identified with the lepton number violating (LNV) MSSM flat direction LLe appearing in the super potential as [39,40]:

$$\mathcal{W}_{\not R}^{\mathrm{MSSM}} \supset \frac{1}{2}\epsilon_{ab}\lambda^{ijk}\mathbf{L}_i^a\mathbf{L}_j^b\bar{\mathbf{e}}_k + h.c.$$

where am = 1, 2 are weak isospin indices and flatness constraint requires I

$$\Gamma\left(\boldsymbol{\Phi}^+ \to \nu_i l_j^+\right) = \frac{m_{\boldsymbol{\Phi}}}{16\pi^2}\bar{\mathbf{F}}_{\mathbf{ij}},$$

$$\Gamma\left(\boldsymbol{\Phi}^- \to \nu_i l_j^-\right) = \frac{m_{\boldsymbol{\Phi}}}{16\pi^2}\bar{\mathbf{B}}_{\mathbf{ij}}$$

where the bilinear functions $\bar{\mathbf{F}}_{\mathbf{ij}}$ and $\bar{\mathbf{B}}_{\mathbf{ij}}$, can be expressed as:

$$m_{\boldsymbol{\Phi}}\bar{\mathbf{F}}_{\mathbf{ij}} = \frac{1}{3}\sum_k m_{\phi_k}\mathbf{F}_{\mathbf{ijk}},$$

$$m_{\boldsymbol{\Phi}}\bar{\mathbf{B}}_{\mathbf{ij}} = \frac{1}{3}\sum_k m_{\phi_k}\mathbf{B}_{\mathbf{ijk}}.$$

Here me be the flavour independent inflation mass and make represents the k-th flavour deependent mass of the constituent Ak. Additionally, the expression for trilinear functions Fink and Bik are explicitly mentioned in Appendix A. To understand the thermal history of the universe from our model, it is convenient to express the decay width in terms of the Hubble parameter during the epoch of reheating as

$$m_{\boldsymbol{\Phi}}\sum_{ij}(\bar{\mathbf{F}}_{\mathbf{ij}} + \bar{\mathbf{B}}_{\mathbf{ij}}) = 16\pi^2\Gamma_\phi(T_r) = 48\pi H_{rh} \quad (13)$$

where $\Gamma_\phi(T_r)$ be the total decay width. In the present context the Hubble parameter during reheating is defined through the modified Friedman equation as given by:

$$H_{rh} \approx \sqrt[4]{\frac{\rho_{rh}}{3M_p^2\theta}}. \quad (14)$$

Here ρ_{rh} be the energy density during reheating. Hence using Eq. (13) the reheating temperature can be expressed as:

$$T_r = \sum_{ij}\sqrt[4]{\frac{5M_p^2\theta m_{\boldsymbol{\Phi}}^4}{294912\pi^6 N_*}(\bar{\mathbf{F}}_{\mathbf{ij}} + \bar{\mathbf{B}}_{\mathbf{ij}})} = \sqrt[4]{\frac{5M_p^2\theta}{18432\pi^2 N_*}\Gamma_\phi} \quad (15)$$

where $N_* (= N_B + \frac{7}{8}N_F)$ be the effective number of relativistic degrees of freedom. Usually $N_* \approx 228.75$ for all MSSM degrees of freedom. Recent observational data from PLANCK suggests an upper-bound on the reheating temperature [26,44–46]:

$$T_r \leqslant 6.654 \times 10^{15}\sqrt[4]{\frac{r_*}{0.12}}\ \mathrm{GeV} \quad (16)$$

where r_* be the tensor-to-scalar ratio at the pivot scale of momentum k_*. Consequently the upper-bound of total decay width during reheating is given by:

$$\Gamma_\phi = \sum_{ij}\frac{m_{\boldsymbol{\Phi}}}{16\pi^2}(\bar{\mathbf{F}}_{\mathbf{ij}} + \bar{\mathbf{B}}_{\mathbf{ij}}) \leqslant 2.772 \times 10^{-3}\sqrt[4]{\frac{3072\pi^2 M_p^2 N_* r_*}{\theta}} \quad (17)$$

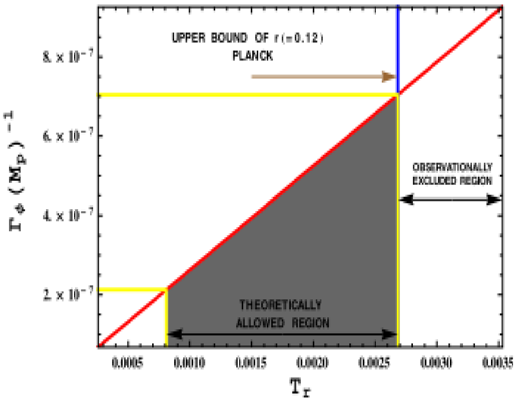

Fig. 1. Variation of total decay width Γφ with respect to reheating temperature (Tr). The dark grey shaded area shows the theoretically allowed region which lies within the upper bound of the tensor-to-scalar ratio (r 0.12) at the momentum pivot scale k= 0.002 Mpc−1 represented by a blue vertical line in light of PLANCK data. We have also pointed the excluded parameter space for the tensor-to-scalar ratio within the range 0.12 < r 0.36 by imposing the constraint from PLANCK data. For the numerical estimation in the present context we have used, G1 = 1, G2 = 0.5, G3 = 2, G4 = 1, G5 = −0.5 and B1 = 2M2 P, B2 = 2, B3 = 1, B4 = 3, B5 = 2M4 P. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.) where the stringent constraint on the slept-on masses and soft SUSY breaking trilinear coopling are meek = 300 GeV and ink |ink| 2 = 1.7786 at the GUT scale, which are obtained by solving the one-loop renormalization group equation in DR scheme [41]. In Fig. 1 we have shown the behaviour of the total decay width as a function of reheating temperature by imposing the observational constraints in light of PLANCK data. Additionally, we have also pointed the theoretically allowed region obtained from the model as well as the observationally excluded parameter space. It is important to note that saturating the upper-bound on r ∼ 0.12 would yield a large reheating temperature of the universe. In this case, the gravitino abundance is compatible with the latest observational/phenomenological bound on dark matter, provided the gravitino mass, m3/2 ∼ O (100) eV, see [47]. The light gravitino is a very interesting candidate for dark matter among various other candidates, since the gravitino itself is a unique and inevitable prediction of supergravity (SUGRA) theory. This prediction is very much interesting, since we can test the gravitino dark matter hypothesis at LHC or through any other indirect probes. In fact, if we had late time entropy production after the decoupling time of the gravitino, the mass of the gravitino dark matter may be raised up to a few keV. Moreover, the gravitino dark matter with a mass in the range m3/2 ∼ O (1–10) keV serves as the warm dark matter candidate which has recently been invoked as possible solutions to the seeming discrepancies between the observation and the simulated results of the galaxy formation based on the cold dark matter (CDM) scenario [48–51]. See [47] for the details of such scenario. Additionally, the gravitino mass of this order is also favoured from several other phenomenological issues, the interesting parameter space for the gauging masses at the LHC, and the solution to the well-known μ-problem [52]. By assuming such a phenomenological prescription perfectly holds good in our prescribed string theoretic setup let us start with a situation where the inflation field starts oscillating when the inflationary epoch ends at a cosmic time t = toss ft and the reheating phenomenology is described by the Boltzmann equation [24]

$$\dot{\rho}_r + 4H\rho_r = \Gamma_\phi \rho_\phi, \tag{18}$$

where $H \approx \sqrt[4]{\frac{\rho_r+\rho_\phi}{3M_P^2\theta}}$. Here ρ_r and ρ_ϕ represent the energy density of radiation and inflaton respectively. Assuming $\Gamma_\phi \gg H$ from we get

$$\rho_\phi = \frac{\rho_{\phi_0}}{x^4}\exp\left[-\Gamma_\phi(t - t_{osc})\right] \tag{19}$$

where $\rho_{\phi_0} = \beta_0$ (the energy scale of DBI Galileon inflation as appearing in Eq. (4)) and additionally we introduce a new parameter "x" defined as:

$$x := \frac{a}{a_{osc}} = \left[1 + H_{osc}(t - t_{osc})\right] \tag{20}$$

with $H_{osc} = (\frac{\rho_{osc}}{3M_P^2\theta})^{1/4}$. For $t \leqslant \Gamma_\phi^{-1}$ the exact solution of Eq. (18) can be written as

$$\rho_r = \frac{1}{x^4}\left[\rho_{osc} - \rho_{\phi_0}\exp\left(-\frac{(x-1)\Gamma_\phi}{H_{osc}}\right)\right]. \tag{21}$$

Finally, we are interested in to compute the thermal dark matter gravitino relic abundance produced by the scattering of the inflaton decay products. To serve this purpose we start with the master equation of gravitino phenomenology as obtained from *Boltzmann equation* is given by [24]:

$$\left(\frac{d}{dt} + 3H\right)n_{3/2} = \langle\Sigma_{total}|v|\rangle n^2 - \frac{m_{3/2}n_{3/2}}{\langle E_{3/2}\rangle \tau_{3/2}}, \tag{22}$$

where n = ζ (3)T 3 π2 is the number density of scatterers (bosons in thermal bath) with ζ (3) = 1.20206 .... Here Total is the total scattering cross section for thermal gravitino produceton, v is the relative velocity of the incoming particles with v = 1 where ⋯ represents the thermal average. The factor m3/2 E3/2 represents the averaged Lorentz factor which comes from the decay of gravitinos can be neglected due to weak interaction. For the gauge group SU (3)C ⊗ SU (2)L ⊗ U (1)Y the thermal gravitino production rate is given by

$$\langle\Sigma_{total}|v|\rangle = \frac{\tilde{\alpha}}{M_P^2}$$
$$= \frac{3\pi}{16\zeta(3)M_P^2}\sum_{i=1}^{3}\left[1 + \frac{M_i^2}{3m_{3/2}^2}\right]C_i g_i^2 \ln\left(\frac{K_i}{g_i}\right), \tag{23}$$

where $i = 1, 2, 3$ stands for the three gauge groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ respectively. Here M_i represent gaugino mass parameters and $g_i(T)$ represents gaugino coupling constant at finite temperature (from MSSM RGE) [24,25]:

$$g_i(T) \simeq \frac{1}{\sqrt{\frac{1}{g_i^2(M_Z)} - \frac{b_i}{8\pi^2}\ln\left(\frac{T}{M_Z}\right)}} \tag{24}$$

with $b_1 = 11$, $b_2 = 1$, $b_3 = -3$. Here C_i and K_i represents the constant associated with the gauge groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ with $C_1 = 11$, $C_2 = 27$, $C_3 = 72$ and $K_1 = 1.266$, $K_2 = 1.312$, $K_3 = 1.271$ [24,25].

Further re-expressing Eq. (22) in terms of the parameter "x" and imposing the boundary condition T˙ = 0 at maximum energy density or (x = mix) the thermal gravitino dark matter relic abundance is given by

$$\Omega_{3/2}(x) = \frac{n_{3/2}(x)}{s(x)}$$
$$= \frac{45}{2\pi^2 x^3 N_\star T^3(x)}\Bigg[\frac{C_3}{4}\sqrt{\frac{C_2 + C_1(x-1)}{x^4}}\Bigg\{C_1(2-5x) - 2C_2$$
$$+ \frac{3C_1^2 x^2}{\sqrt{(C_1 - C_2)(C_2 + C_1(x-1))}}\tan^{-1}\left(\sqrt{\frac{C_2 + C_1(x-1)}{C_1 - C_2}}\right)\Bigg\}$$
$$+ \frac{1}{3x_{max}^2}\{C_3(C_2 + C_1(x_{max} - 1))\}^{\frac{3}{2}}$$
$$- \frac{C_3}{4}\sqrt{\frac{C_2 + C_1(x_{max}-1)}{x_{max}^4}}\Bigg\{C_1(2 - 5x_{max}) - 2C_2$$
$$+ \frac{3C_1^2 x_{max}^2}{\sqrt{(C_1 - C_2)(C_2 + C_1(x_{max}-1))}}\tan^{-1}\left(\sqrt{\frac{C_2 + C_1(x_{max}-1)}{C_1 - C_2}}\right)\Bigg\}\Bigg], \quad (25)$$

where the entropy density is given by $s(x) = \frac{2\pi^2}{45}N_\star T^3(x)$. Here the temperature can be expressed in terms of the tensor-to-scalar ratio and the parameter "x" as:

$$T(x) = \frac{1}{\sqrt{\pi}}\sqrt[4]{\left[\frac{30}{N_\star x^4}\left(\rho_{osc} - \rho_{\phi_0}\left\{1 - \frac{0.48(x-1)}{H_{osc}}\sqrt[4]{\left(\frac{N_\star M_P^2}{\theta}\right)}\right\}\right)\right]} \quad (26)$$

and we also introduce new sets of parameters defined as:

$$C_1 = \frac{\Gamma_\phi \rho_{\phi_0}}{H_{osc}}, \quad C_2 = \rho_{osc} - \rho_{\phi_0},$$
$$C_3 = \frac{30\sqrt{10}\bar{\alpha}\zeta^2(3)}{H_{osc}\pi^7 M_P^2 N_\star^{3/2}}, \quad x_{max} = \frac{4}{3} - \frac{4C_2}{9C_1 H_{osc}^2}. \quad (27)$$

In this paper we introduce the lipogenesis scenario in presence of DBI Galileo which has the following remarkable phenomenological features: • In Fig. 1, the theoretically allowed region shows that the reheating temperature for DBI Galileo is high enough and lies around the GUT scale (1016 GeV). This is the first Obserovation we have made from our analysis in the context of DBI Galileo, which is remarkably different from the GR prescribed setup as using GR we can probe up to 1010 GeV. Such high values of the reheating temperature implies that the obtained value of the tensor-to-scalar rayTio from the DBI Galileo inflationary set up lies within a wide range: 2.4 × 10−3 < r< 0.12, at the pivot scale of momentum k ∼ 0.002 Mpc−1, which confronts well the Planck data. If the signatures of the primordial gravity waves will be detected at present or in near future then the consistency between the high reheating temperature and gravity waves can be directly verified from our prescribed model using El

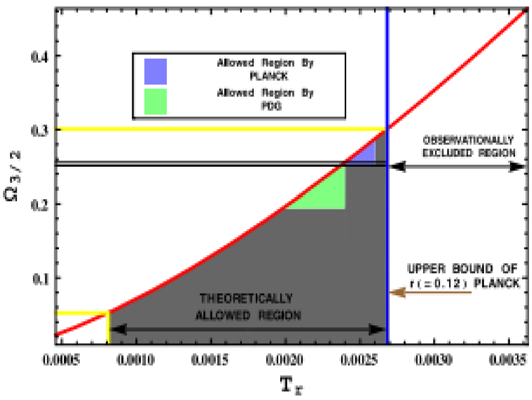

Fig. 2. Variation of gravitino dark matter relic density parameter (Ω3/2) with respect to reheating temperature (Tr). The dark grey shaded region shows the theoretically allowed region which lies within the upper bound of the tensor-toscalar ratio (r 0.12) at the momentum pivot scale k = 0.002 Mpc−1 represented by a blue vertical line

in light of PLANCK data. We have explicitly shown the observationally allowed region of Ω3/2 by imposing the constraints from the PLANCK data. Further we have also pointed the constraint parameter space obtained from the PDG catalo. Most importantly, the overlapping region within the range 0.245 < Ω3/2 < 0.250 shown by the black strip satisfies both the constraints obtained from PLANCK and PDG data. For the numerical estimation in the present context we have used, G1 = 1, G2 = 0.5, G3 = 2, G4 = 1, G5 = −0.5 and B1 = 2M2 P, B2 = 2, B3 = 1, B4 = 3, B5 = 2M4 P. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

- In Fig. 2 we have explicitly shown the behaviour of gravitino relic abundance with respect to reheating temperature in light of PLANCK and PDG data. The overlapping region within the range $0.245 < \Omega_{3/2} < 0.250$ satisfies both the dark matter constraints obtained from PLANCK and PDG data as given by [53,54]:

$$\Omega_{\rm DM}^{\rm PLANCK} = 0.26 \pm 0.01$$
$$\Omega_{\rm DM}^{\rm PDG} = 0.22 \pm 0.03. \quad (28)$$

In the present article we have studied cosmological consequences of reheating and dark matTer phenomenology in the context of DBI Galileo on the background of low energy effective supergravity framework. We have engaged ourselves in investigating for the effect of perturbtie reheating by imposing the constraints from primordial gravitational waves in light of the PLANCK data. Further we have established a cosmological connection between thermal graveIino dark matter relic abundance, reheating temperature and tensor-to scalar ratio in the present context. To this end we have explored the model dependent features of thermal relic gravitino abundance by imposing the dark matter constraint from PLANCK + PDG data, which is also consistent with the additional constraint associated with the upper bound of tensor-to-scalar ratio r0.002 0.12 obtained from PLANCK data.

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Appendix A

In Eqs. (11,13,15,17) the trilinear functions are given by:

$$\mathbf{F_{ijk}} = |\lambda_{ijk}|^2\left[2 + \frac{9g^4}{4c_w^4}\left(\frac{1}{4} + 2s_w^4 - s_w^2\right)I_1^2\left(m_{\phi_k}^2, 0, 0\right)\right]$$
$$\mathbf{B_{ijk}} = |\lambda_{ijk}|^2\left[2 + \frac{9g^4}{4c_w^4}\left(\frac{1}{4} + 2s_w^4 + s_w^2\right)I_1^2\left(m_{\phi_k}^2, 0, 0\right)\right] \quad (29)$$

where the integral $I_1(m_k^2, m_i^2, m_j^2)$ is defined as

$$I_1\left(m_k^2, m_i^2, m_j^2\right) = -i\int_0^1\int_0^1 dx\,dy\left[\gamma_E + \frac{1}{2} + \frac{N(x,y)}{2Q^2(x,y)} + \ln\left[\frac{Q^2(x,y)}{4\pi\mu^2}\right]\right] \quad (30)$$

with

$$N(x,y) = x(1-x)m_j^2 + y(1-y)m_i^2 + \frac{1}{2}\left[(1-x-y)\left(m_k^2 - m_i^2 - m_j^2\right)\right]$$
$$Q^2(x,y) = x^2m_j^2 + y^2m_i^2 - xy\left(m_k^2 - m_i^2 - m_j^2\right) + (1-x-y)m_z^2. \quad (31)$$

In Eq. (30) $\gamma_E = 0.5772$ is the *Euler–Mascheroni constant* originating in the expansion of the gamma function. Here $c_w = \cos\theta_w$, $s_w = \sin\theta_w$ (where θ_w = Weinberg angle) and m_z be the mass of the Z boson.

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